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Note

Correction to a Dimensional Perturbation Calculation of the Contraction Coefficient

In Ref. [1] Garabedian describes a method for estimating certain constants in axially symmetric cavitational flows and jets. The three-dimensional model is treated as a perturbation of the (known) two-dimensional case with m as the perturbing parameter, where $m + 2$ denotes the number of spacial dimensions. In this note we report a correction to the calculation of the contraction coefficient for the vena contracta.

It is conjectured that the ratio of radius (or half-width) of the jet at infinity to the radius of the aperture, X/Y, may be expressed in $m + 2$ dimensions as a power series expanded around $m = 0$. Thus

$$X(m)/Y(m) = [X(0)/Y(0)] + a_1 m + a_2 m^2 + \cdots .$$

Regarding m as a parameter in the equations for the vena contracta, Garabedian is able to calculate

$$X(-1)/Y(-1) = 0,$$
$$X(\infty)/Y(\infty) = 1$$

and, of course,

$$X(0)/Y(0) = \Pi/(\Pi + 2).$$

Furthermore, the derivative $\partial Y/\partial m$, evaluated at $m = 0$ for constant $X = 1$, is expressed as follows;

$$\frac{\partial Y}{\partial m} = -\frac{2}{\Pi} \int_0^1 \int_0^\infty \left[\frac{3 + a^2}{(1 + a^2)^2} + \frac{\tan^{-1} a}{a}\right] \cdot \tan^{-1} \frac{2\gamma a}{4\Pi a^2 + \Pi b^2 + \gamma b} \, db \, da,$$

where γ is a function of a and b (see [1] for details). This integral was evaluated numerically in [1] and the value is quoted as

$$\partial Y/\partial m = -0.650544.$$

Interpolating the above data with a cubic polynomial in $\delta = m/(m + 2)$ leads to the expression

$$X/Y = 0.6110 + 0.4857\delta - 0.1110\delta^2 + 0.0143\delta^3$$

as an approximation to the power series. For three dimensions, $\delta = 1/3$ and the result is

$$X(1)/Y(1) = 0.7611$$

leading to the estimate of the contraction coefficient C

$$C = [X(1)/Y(1)]^2 = 0.5793.$$

An indication of the accuracy of the computation is provided by the decrease in the magnitudes of the terms of the polynomial. An error of 1/10 percent is estimated.

Using the larger CDC 6600 computer at New York University we re-evaluated the integral for $\partial Y/\partial m$ and found that a more accurate value is

$$\partial Y/\partial m = -0.7072 \pm 0.0003,$$

leading to the cubic polynomial

$$X/Y = 0.6110 + 0.5279\delta - 0.1110\delta^2 - 0.0279\delta^3$$

which fits the data at $m = -1$, 0, and ∞ as before.

The revised computation of C for $m = 1$ is then

$$C = 0.7737^2 = 0.5985.$$

A glance at the magnitudes of the terms in the polynomial now reveals that the error estimate must also be revised upwards. Consideration of this data in the light of other computations of C (cf. [2]) leads us to conclude that the finite-difference calculations are probably more reliable; they yield the value

$$C = 0.59135,$$

and the error is quoted as ± 0.00004.

References

1. P. R. Garabedian, *Pac. J. Math.* **6** (1956), 611.
2. E. Bloch, "A Finite Difference Method for the Solution of Free Boundary Problems," AEC Computing and Applied Mathematics Center, Courant Inst. Math. Sci., New York Univ., NYO-1480-116, 1969.

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Arthur D. Snider

Department of Mathematics,
University of South Florida,
Tampa, Florida 33620

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